Available online at www.sciencedirect.com
SCIENCE @ DIRECT®
Computers & Operations Research 33 (2006) 520–534
computers & operations research
www.elsevier.com/locate/cor

Splitting-type method for systems of variational inequalities

I.V. Konnov

Department of Applied Mathematics, Kazan University, ul. Kremlevskaya, 18, Kazan 420008, Russia

Available online 28 July 2004

Abstract

We consider a system of variational inequalities with multivalued mappings, which can be viewed as an extension of constrained primal–dual variational inequalities. We propose to solve this problem with the help of a class of combined relaxation and splitting methods. We establish a convergence result for these methods for the case where the dual system is solvable. We also give several examples of applications to saddle point problems in optimization, Nash equilibrium problems in game theory, and economic equilibrium problems.

Keywords: System of variational inequalities; Multivalued mappings; Splitting method

1. Introduction

Let U be a convex and closed subset of a real l-dimensional Euclidean space R^l, $G : U \to R^l$, a single-valued mapping, and $F : U \to \Pi(R^l)$, a multivalued mapping (Here and below $\Pi(S)$ denotes the family of all subsets of a set S). Then one can define the following *variational inequality problem* (VI for short): find an element $u^* \in U$ such that

$$\exists f^* \in F(u^*), \quad \langle G(u^*) + f^*, u - u^* \rangle \geqslant 0 \quad \forall u \in U. \tag{1}$$

Usually, convergence of iterative methods to a solution of this very general problem needs certain monotonicity assumptions on the mappings G and F; see e.g. [1,2]. Moreover, there exist few approaches for constructing convergent methods for such VIs without additional integrability or strict monotonicity assumptions. On the one hand, most simple methods, such as averaging and iterative regularization methods (see [3,4]) have low convergence due to the divergent series stepsize rules. On the other hand, more complicated methods such as proximal and level ones [5–9] involve auxiliary procedures which are very

E-mail address: igor.konnov@ksu.ru (I.V. Konnov).

0305-0548/$ - see front matter © 2004 Elsevier Ltd. All rights reserved.
doi:10.1016/j.cor.2004.06.019